

Stochastic ratcheting of two dimensional colloids : Directed current and dynamical transitions

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We present results of molecular dynamics simulations for two-dimensional repulsively interacting colloids driven by a one dimensional asymmetric and commensurate ratchet potential, switching on and off stochastically. This drives a time-averaged directed current of colloids, exhibiting resonance with change in ratcheting frequency, where the resonance frequency itself depends non-monotonically on density. Using scaling arguments, we obtain analytic results that show good agreement with numerical simulations. With increasing ratcheting frequency, we find *non-equilibrium re-entrant transitions* between solid and modulated liquid phases.

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A flashing ratchet refers to a time-averaged directed motion of Brownian particles under the influence of an spatially periodic and asymmetric potential, with the potential height varying with time, either deterministically or stochastically [1–4]. Stochastic ratcheting has been studied extensively, in the context of active dynamics of molecular motors [5–8], dynamics of colloidal dispersion in electrical [9–11], magnetic [12, 13] or optical drive [14, 15], as a mechanism of particle segregation [16–18], transport of cold atoms in optical lattice [19], and in the motion of flux quanta [20, 21]. While a large body of work has been concentrated on the ratcheting of individual particles, fewer studies focused on the effects of interaction [22–26]. Recent studies of two dimensional (2D) paramagnetic particles under one dimensional (1D) magnetic ratchets observed relation between overall dynamics and local particle coordination numbers [13].

In colloidal suspensions, ratchet-like directed motion of particles have been achieved using suitable laser potentials [14, 15]. Confinement and laser trapping in colloids, on the other hand, is known to give rise to interesting mechanical properties and phase transitions [27–32]. Coupling 2D interacting colloids to a 1D time-independent spatially periodic potential with periodicity commensurate with the mean particle separation, leads to the phenomena of laser induced freezing (LIF) and re-entrant melting with increase in the potential strength. This was demonstrated in experiments using standing wave pattern of interfering laser beams [31, 32], and was understood in terms of a dislocation unbinding theory [9, 33].

We consider transport of a 2D system of particles interacting via soft-core repulsion and driven by an 1D asymmetric flashing ratchet, using molecular dynamics (MD) simulations in the presence of a Langevin heat bath. The ratcheting potential breaks time-reversal symmetry and generates an average directed current along the direction of ratcheting (Fig. 1(a)). We choose a periodicity of the potential commensurate with the inter particle separation. At switching frequencies much faster than the

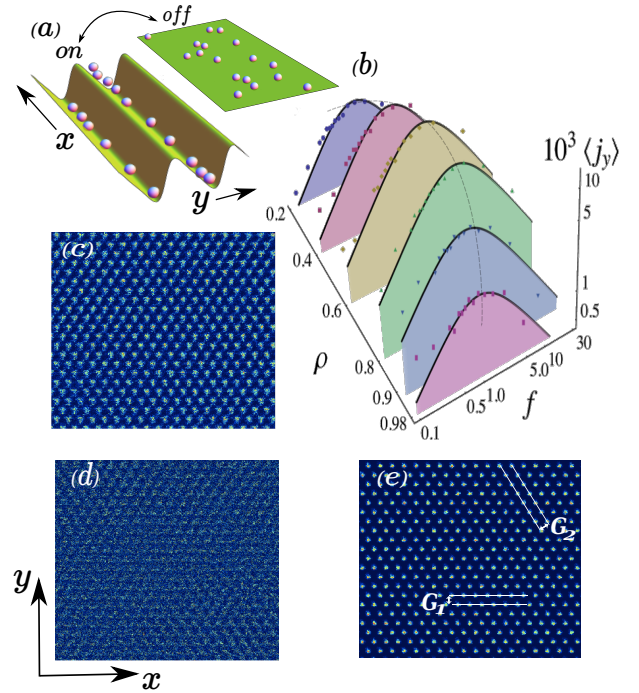


FIG. 1. (Color online) (a) Schematic of 2d colloids in 1D asymmetric ratchet potential periodic in y - and constant in x - direction. The arrow between the (green) corrugated and flat surfaces denote switching of external potential between *on* and *off* state with rate f . (b) Time-averaged directed current along y -direction $\langle j_y \rangle$ as a function of frequency f , and density ρ . The dashed line indicates variation of maximal current with density. (c) – (e): Superimposed positions of 10^3 uncorrelated configurations, from center of mass coordinates, at a density $\rho = 1.0$ with ratcheting frequencies (c) $f = 0.11$, (d) $f = 1.67$ and (e) $f = 10$. The color code denotes local density of points from red/light (high) to blue/dark (low). The reciprocal lattice vectors $\mathbf{G}_{1,2}$ and the corresponding lattice planes are indicated in (e).

intrinsic relaxation times, the time scale required for particles to relax over a single valley of the external ratchet potential, the system experiences a time-averaged effective periodic potential, which in the limit of weak asymmetry is expected to lead to a situation similar to that of LIF. However, at intermediate switching frequencies the system is driven out of equilibrium and carries an averaged directed current. We present the transport properties, and relation between structure and transport.

In the 2D ratchet system that we study, averaged directed current shows non-monotonic variation with density and ratcheting frequency, with the maximal current achieved at their intermediate values. The behavior differs significantly in detailed functional dependence from 1D ratchet. Using scaling arguments, we derive expressions for the directed current which fully capture the simulation results. Our study on 2D ratchet reveals two fascinating properties which are unlike 1D ratchet: (i) with increasing ratcheting frequency we find *re-entrant non-equilibrium phase transitions* between solid and modulated liquid phases, as averaged directed current shows non-monotonic variation, (ii) crossover from ballistic to diffusive transport with density, captured by a non-monotonic density-dependence of resonance frequency. Our predictions are amenable to verification in experiments on, e.g., sterically stabilized colloids driven by suitably tunable optical or magnetic ratchets [13, 14].

Model: As a model colloid, we consider a system of purely repulsive particles interacting via a shifted and truncated soft-core potential $\beta U(r) = (\sigma/r)^{12} - 2^{-12}$ with a cutoff distance $r_c = 2\sigma$, so that $\beta U(r) = 0$ for $r > r_c$. Here $k_B T = 1/\beta$ and σ set the energy and length scales, respectively. The asymmetric ratchet potential $U_{\text{ext}}(y, t) = V_0(t) [\sin(2\pi y/\lambda) + \alpha \sin(4\pi y/\lambda)]$, where $V_0(t)$ switches between U_0 and 0 with a switching rate f , which we also refer to as frequency. We use the asymmetry parameter $\alpha = 0.2$ (see Fig. 1(a)). In all our simulations we set $\beta U_0 = 1$. The external potential is kept commensurate to the density of the particles, such that $\lambda = a_y$, with the separation between consecutive lattice planes $a_y = \sqrt{3}a/2$ in a triangular lattice at a density $\rho = 2/\sqrt{3}a^2$. MD simulations are performed using the standard leap-frog algorithm [35] with a time step $\delta t = 0.001\tau$, where $\tau = \sigma\sqrt{m/k_B T}$ is the characteristic time scale. We choose the mass of the particles $m = 1$, and set the temperature $T = 1.0\epsilon/k_B$ by using a Langevin thermostat [36] with an isotropic friction $\gamma = 1/\tau$. At each time step, a trial move to perform switching of the external potential strength between 0 and U_0 is performed, and accepted with probability $f\delta t$. We used $N = 4096$ particles in our simulations.

The soft-core particles, in the absence of external potential, freezes at a density $\rho^* \approx 1.01$ (see Fig.1(a) in the Supplemental Material [37]). The limit of $\alpha = 0$ and $V_0(t) = U_0$ corresponds to the equilibrium situation of laser induced freezing [33]. At a density close to

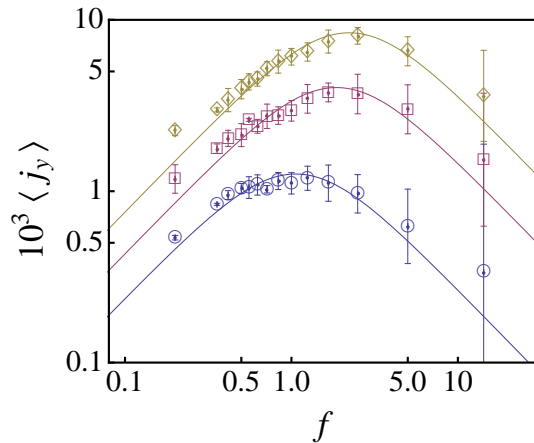


FIG. 2. (Color Online) Average directed current as a function of switching rate f , at particle densities, $\rho = 0.1$ (\circ), 0.5 (\square) and 1.0 (\diamond). The solid lines show fit to Eq. 2.

the liquid-solid transition, the system freezes into a triangular lattice solid (LIF) which remelts into a density modulated liquid with increasing U_0 [32, 33]. In soft-core particles, the LIF with $\beta U_0 = 1$ occurs at $\rho = 0.95$ [9]. Similar freezing transition at this density is observed for a weakly asymmetric ratchet ($\alpha = 0.2$) of strength $\beta U_0 = 1$ in the limit of high switching frequency, much faster than the typical relaxation time, such that the colloids experience an effective periodic potential (see Fig.2(d) in the Supplemental Material [37]). In the other limit of extremely slow switching, the system comes to *quasi-equilibrium* with the instantaneous strength of external potential, and one obtains a slow variation between a modulated liquid and a solid phase. The most interesting dynamics takes place at intermediate frequencies. The ratchet-driven averaged directed current shows resonance with frequency, and non-monotonic variation with density (Fig. 1(b)). At suitable densities, the system shows dynamical re-entrant transition from a soft solid to modulated liquid to solid with increase in ratcheting frequency (Fig. 1(c)–(e) and Fig. 5).

Transport properties: The steady state dynamics is characterized in terms of a space and time-averaged directed current of particles flowing along the direction of ratcheting

$$\langle j_y \rangle = \frac{1}{\tau_m} \frac{1}{L_x L_y} \int_0^{\tau_m} dt \int_0^{L_x} dx \int_0^{L_y} dy j_y(x, y, t) \quad (1)$$

where the time averaging is done over $\tau_m = n t_p$, with $t_p = 1/f$ and n denotes a large number of switching, chosen to be 200 in all our simulations.

For small switching frequencies $f \ll \nu$, the inverse of intrinsic relaxation time, the system is close to thermodynamic equilibrium. The directed current increases as $\langle j_y \rangle \sim f$ starting from zero at $f = 0$ in agreement with linear response [5, 38, 39]. The frequency dependence at

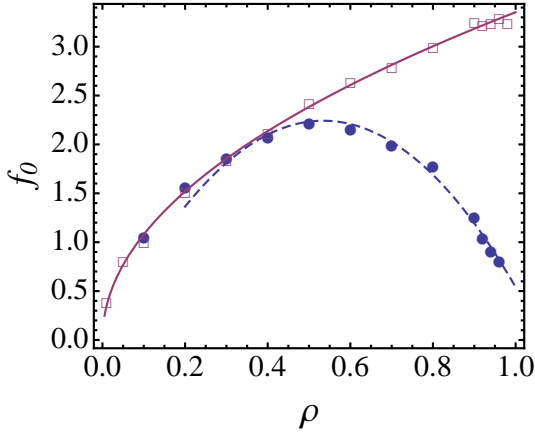


FIG. 3. (Color Online) Resonance frequency f_0 as a function of the density ρ for non-interacting particles (squares), and soft-core particles (circles). The solid line shows ballistic form $f_0 \sim \sqrt{\rho}$, while the dashed line shows diffusive form $f_0 \sim \rho/(1 - \rho/\rho_c)$ with $\rho_c = 1.07$.

high switching rate was calculated earlier using asymptotic expansion to give $\langle j_y \rangle \sim 1/f$ [39, 40].

In our MD simulations of 2D system of soft-disks, we observe the same behavior, viz., $\langle j_y \rangle \sim f$ at low frequency, and $\langle j_y \rangle \sim 1/f$ at very high ratcheting frequencies (Fig. 2). The asymptotic behavior may be captured by the interpolation formula $g(\nu, f) = \nu f / (\nu^2 + f^2)$. We use a simple ansatz $\langle j_y \rangle = \kappa g(\nu, f) \rho v_0$ where κ is a dimensionless proportionality constant, and ρv_0 has the dimension of current with v_0 an intrinsic velocity. As we show below, the form of v_0 and ν allow us to describe the whole density and frequency dependence of directed current. The relation

$$\langle j_y \rangle = \kappa \frac{\nu f}{\nu^2 + f^2} \rho v_0, \quad (2)$$

shows good agreement with simulation results (Fig. 2). The above frequency dependence is obeyed even if the ratcheting wavelength λ is incommensurate with density (see Fig.2(a)-(c) in the Supplemental Material [37]). A similar frequency dependence was recently found for a stochastic pump model of one-dimensional system of interacting particles [41]. Fitting the MD simulation data of Fig. 2 to Eq.(2) we find the resonance frequencies $f = f_0 = \nu$ which show a non-monotonic variation with the mean density of colloids ρ (Fig. 3).

The intrinsic relaxation frequency ν , controlling the behavior of time-averaged dynamics, may arise from a ballistic or diffusive relaxation of the particles over the characteristic length scale λ . We use ratcheting potential commensurate with the density such that $\lambda^2 \sim 1/\rho$ (for treatment using incommensurate potential, see Supplemental Material [37]). For under-damped motion, the ballistic time-scale τ_b for a particle to traverse the potential valley is obtained from the kinematic relation $\lambda \sim (U_0/\lambda)\tau_b^2$, that leads to $\tau_b \sim (\rho U_0)^{-1/2}$. On

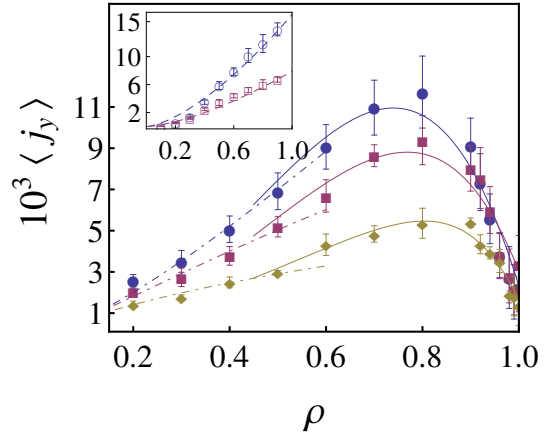


FIG. 4. (Color Online) Average particle flux $\langle j_y \rangle$ as a function of density ρ , for soft-core particles at ratcheting frequencies $f = 1.43$ (\bullet), 0.71 (\blacksquare), 0.36 (\blacklozenge). The dot-dashed lines are fit to Eq.(3) in the regime $\rho < 0.5$ with fitting parameter $\kappa = 0.04, 0.03, 0.02$ for the three data sets respectively. The solid lines are fit to Eq.(4) in the regime $\rho \geq 0.5$ with fitting parameters $\kappa = 0.12, 0.05, 0.02$ and $\rho_c = 1.03, 1.05, 1.03$. Inset: The same quantity for free particles at two different ratcheting frequencies $f = 0.71$ (\circ) and 0.36 (\square). The dashed lines are fit to Eq.(3).

the other hand, the relaxation time in the over-damped diffusive regime is given by $\tau_D = \lambda^2/D \sim (D\rho)^{-1}$. The self diffusion constant D decreases with density for two-dimensional repulsively interacting particles as $D = D_0(1 - \rho/\rho_c)$ [2, 3] (see Fig.1(b) in the Supplemental Material [37]).

In the underdamped case, the velocity scale is set by $v_0^b = \lambda/\tau_b = U_0^{1/2}$. Using this and $\nu = 1/\tau_b$ in this regime, one finds

$$\langle j_y \rangle \simeq \kappa \frac{f U_0}{\rho U_0 + f^2} \rho^{3/2}. \quad (3)$$

The resonance frequency is then $f_0 = (\rho U_0)^{1/2}$. On the other hand, the velocity scale in the over-damped regime may be obtained using the time-scale for free diffusion $1/(\rho D_0)$ over mean inter-particle separation λ , $v_0^D = D_0 \rho^{1/2}$. Thus, using $\nu = 1/\tau_D$ the averaged directed current becomes

$$\langle j_y \rangle \simeq \kappa \frac{f D_0^2}{D_0^2 \rho^2 (1 - \rho/\rho_c)^2 + f^2} \rho^{5/2} (1 - \rho/\rho_c). \quad (4)$$

The corresponding resonance frequency is $f_0 = D_0 \rho (1 - \rho/\rho_c)$.

Our simulations show that the resonance frequency, and therefore the intrinsic relaxation frequency, follows ballistic behavior $f_0 \sim \sqrt{\rho}$ at low densities (and for non-interacting particles), and diffusive behavior $f_0 \sim \rho/(1 - \rho/\rho_c)$ at high densities (Fig. 3). The dynamical behavior changes from ballistic to diffusive with increase in density. This may be understood in terms of what

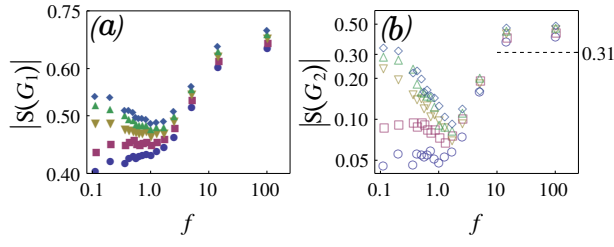


FIG. 5. (Color Online) Amplitude of steady state structure factor, for the reciprocal lattice vectors \mathbf{G}_1 (a) and \mathbf{G}_2 (b), as a function of ratcheting frequency f for densities $\rho = 0.98$ (●), 0.99 (■), 1.00 (▼), 1.01 (▲) and 1.02 (◆).

happens to a directed current in the presence of direction randomizing scattering events. At low densities, the time- and space- averaged motion of a test particle with small number of scattering events remains ballistic on an average. However, at large densities mean free path reduces, and consequently, a large number of scattering events randomizes the direction of motion leading to a predominantly diffusive dynamics.

In Fig. 4 we show the density dependence of the directed current at various switching frequencies. The plots show non-monotonic variation, the low density limit of which is fully captured by Eq.(3), and the high density limit by Eq.(4). Near the density ρ_c , the system gets into a *jammed* state where the directed current vanishes as $\langle j_y \rangle \sim \rho^{5/2}(1 - \rho/\rho_c)$. Note that, the overall density-dependence that we find in 2D ratchet is quite unlike the $\langle j \rangle \sim \rho(1 - \rho)$ behavior of directed current found in repulsively interacting 1D ratchet [26]. The collective dynamics of the 2D ratchet can be further characterized in terms of the density and ratcheting- frequency dependence of longitudinal and transverse diffusivity $D_{x,y}(\rho, f)$ (see Figs 3 and 4 in the Supplemental Information [37]).

Dynamical transitions: The reduction of directed current at high densities and subsequent jamming is associated with freezing of the system into a triangular lattice solid. Our MD simulations showed similar structural transitions are also associated with change in current as a function of ratcheting frequency (Fig. 1(b)), a fully dynamical effect. In Fig. 1(c)–(e), we plot the superimposed positions of 10^3 uncorrelated configurations from the center of mass frame, for a system at a mean density $\rho = 1.0$, and ratcheting frequencies $f = 0.11, 1.67, 10$. This suggests frequency dependent re-entrant *transition* from a triangular lattice solid ($f = 0.11$), to density modulated liquid ($f = 1.67$), to again a triangular lattice solid ($f = 10$) order. Note from Fig. 2 that the modulated liquid at $\rho = 1.0$ and $f = 1.67$ corresponds to the resonance frequency in directed current.

The interplay of structure and dynamics is further quantified with the help of time-averaged steady state structure factor $S(\mathbf{G}) = \langle \frac{1}{N^2} \sum_{i,j} \exp(-\mathbf{G} \cdot (\mathbf{r}_i - \mathbf{r}_j)) \rangle$ with reciprocal lattice vectors $\mathbf{G}_1 = (0, \pm 2\pi/a_y)$ and

$\mathbf{G}_2 = (\pm 2\pi/a, \pm 2\pi/\sqrt{3}a)$ (see Fig. 1(e)). In Fig. 5 we show the frequency dependence of $|S(\mathbf{G}_{1,2})|$ at various densities. The presence of ratcheting potential keeps $|S(\mathbf{G}_1)| > |S(\mathbf{G}_2)|$ corresponding to stronger density modulation in the y -direction. The non-monotonic variation of $|S(\mathbf{G}_1)|$ with frequency quantifies a reduction followed by an increase in this density- modulation. At very high frequencies, the solid- order parameter $|S(\mathbf{G}_2)| > 0.31$ for densities $\rho \gtrsim 0.96$, signifying freezing into a triangular lattice structure (see Figs 1(a) and 2(d) in Supplemental Material [37]), reminiscent of LIF transition [9]. The solid order parameter $|S(\mathbf{G}_2)|$ at densities $\rho \geq 1$ shows significant non-monotonic variation with frequency, pointing to a dynamical re-entrant *transition* from a solid to modulated liquid to solid phase. Thus ratcheting frequency provides a means to structural control during transport, and may be utilized in experiments.

Summary and outlook: Our study on a 2D system of soft-core particles under 1D ratchet drive, have shown interesting relation between transport properties and structural phases. Using scaling arguments we obtained the density and ratcheting frequency dependence of averaged directed current $\langle j_y \rangle$, which fully captured the simulation results. The resonance frequency of $\langle j_y \rangle$ showed a curious cross-over from ballistic to diffusive behavior with increasing density, related to reduction of mean free path. Within a range of densities, we found a *dynamical re-entrant transition* from solid- to modulated liquid- to solid- phase with increasing ratcheting frequency. The fact that ratcheting frequency provides a control over both the emergent directed current and structural phases, may have useful applications.

Our predictions may be verified in experiments on repulsively interacting colloids confined within glass plates, e.g., using magnetic ratcheting [13], or optical ratcheting [14] in a suitably modified 2D laser trapping setup [32]. For example, polystyrene beads have density 1.05 g/cm^3 , i.e., a bead of diameter $\sigma \approx 5 \mu\text{m}$ have mass $m \approx 6.9 \times 10^{-14} \text{ Kg}$. Given $k_B T = 4.2 \times 10^{-21} \text{ Nm}$ at room temperature, the unit of time $\tau = \sigma \sqrt{m/k_B T} \approx 0.02 \text{ s}$. Thus the dimensionless frequency range of $f = 0.1$ to 100 studied here, corresponds to a range of 5 Hz to 5 KHz, and the resonance at $f_0 \approx 1$ means a frequency of 50 Hz.

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Supplementary Information:

Soft-core particles – in the absence of external potential

The phase behavior and dynamics of repulsively interacting colloidal suspensions have been extensively studied in literature [1–3]. The two-dimensional fluid is known to undergo a freezing transition into a triangular lattice solid, arguably via a hexatic phase, with increasing density [4–7]. Without going into the intricacies of identifying the hexatic [1, 7], to determine the fluid-solid transition point we calculate the solid- order parameter $|S(G_2)|$ of the soft-disk system, in the absence of any external driving, as a function of the density of the system, depicted in fig. 6(a). At phase transition, $\rho^* = 1.01$, $|S(G_2)|$ shows a discontinuous increase with density (Fig.6(a) shows MD simulation result of 4096 particles). At this point the order-parameter jumps increase to a value $|S(G_2)| = 0.31$. In all our simulations, even in the presence of ratchet-driving, we identify a phase transition to a solid phase whenever $|S(G_2)|$ crosses the value 0.31. Note that this value is close to the phase transition criterion of $|S(G_2)| = 0.35$ used in recent literature [8]. As depicted in the inset of Fig.6(a), the pressure versus density behavior shows non-monotonicity signifying a phase-coexistence of a solid with density $\rho_s = 1.01$ and liquid with $\rho_l = 0.99$ [1].

In the overdamped regime, the internal relaxation is due to diffusive motion of particles. The density dependence of diffusivity $D(\rho)$ in soft-disk particles has been previously studied by Lahtinen *et. al* [2, 3] and exhibits a linear dependence on ρ . Using our MD simulations, we studied the same to obtain the linear density dependence $D = D_0(1 - \rho/\rho_c)$, where D_0 is the diffusivity of a noninteracting system, and the fitted value $\rho_c = 1.04$ (Fig.6(b)).

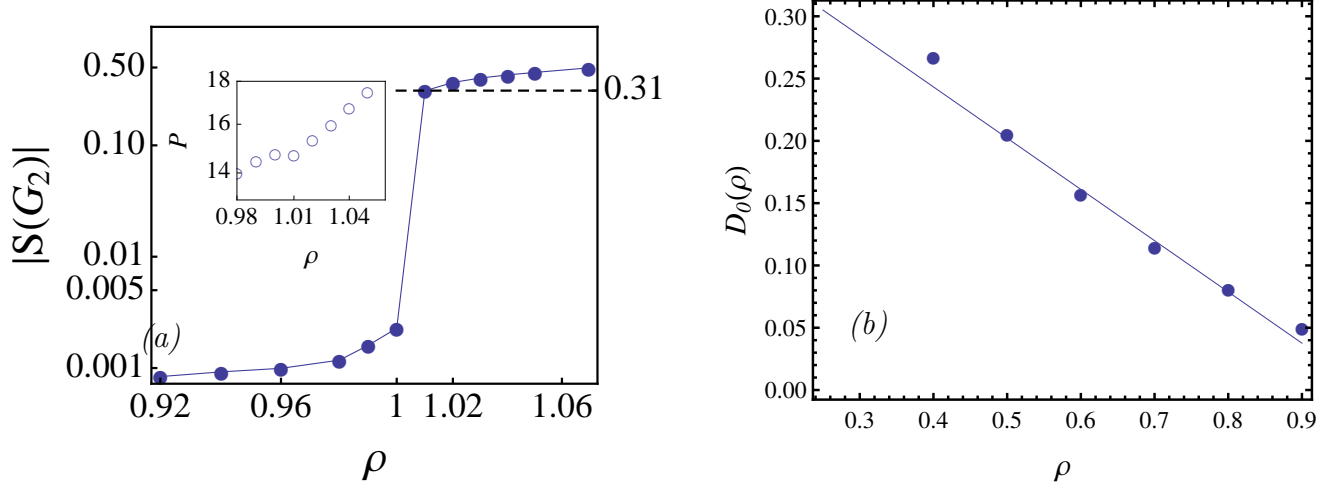


FIG. 6. (a) Plot of order parameter against density for a free system. The discontinuity of the order parameter occurs at a density near $\rho^* = 1.01$. The inset shows pressure versus density behavior and is close to Ref. [1]. (b) Plot of the diffusion coefficient of the free system, averaged over both directions, against the density of the suspension. The solid line is a linear fit of $D = D_0(1 - \rho/\rho_c)$ to the data with $\rho_c = 1.04$.

Soft-core particles – under stochastic ratcheting

In order to study the effect of stochastic ratcheting, the system was evolved under the potential $\beta U_{\text{ext}}(y, t) = \beta V_0(t) [\sin(2\pi y/\lambda) + \alpha \sin(4\pi y/\lambda)]$, with $\beta V_0(t)$ switching between 0 and 1, and asymmetry parameter $\alpha = 0.2$. The switching is done stochastically with a rate f . During our MD simulation, at each time step the value of the external potential $V(t)$ is switched between 0 and 1 with a probability $f\delta t$, and left unaltered with a probability $(1 - f\delta t)$, where δt is the size of integration time-step. We have always waited for the system to reach steady state before collecting data presented in all our analysis.

In this section, we first focus on systems driven by a ratchet of fixed periodicity $\lambda = 1\sigma$. The driving potential is not commensurate with the density, unlike the system discussed in the main text. The impact on the non-interacting system itself is different, as λ is no more a function of density. The integrated directed current $\langle j_y \rangle$ exhibits resonance at a fixed value of the frequency of $f_0 = 3.5$, independent of densities (see Fig. 7 (a)). The ballistic time-scale τ_b to

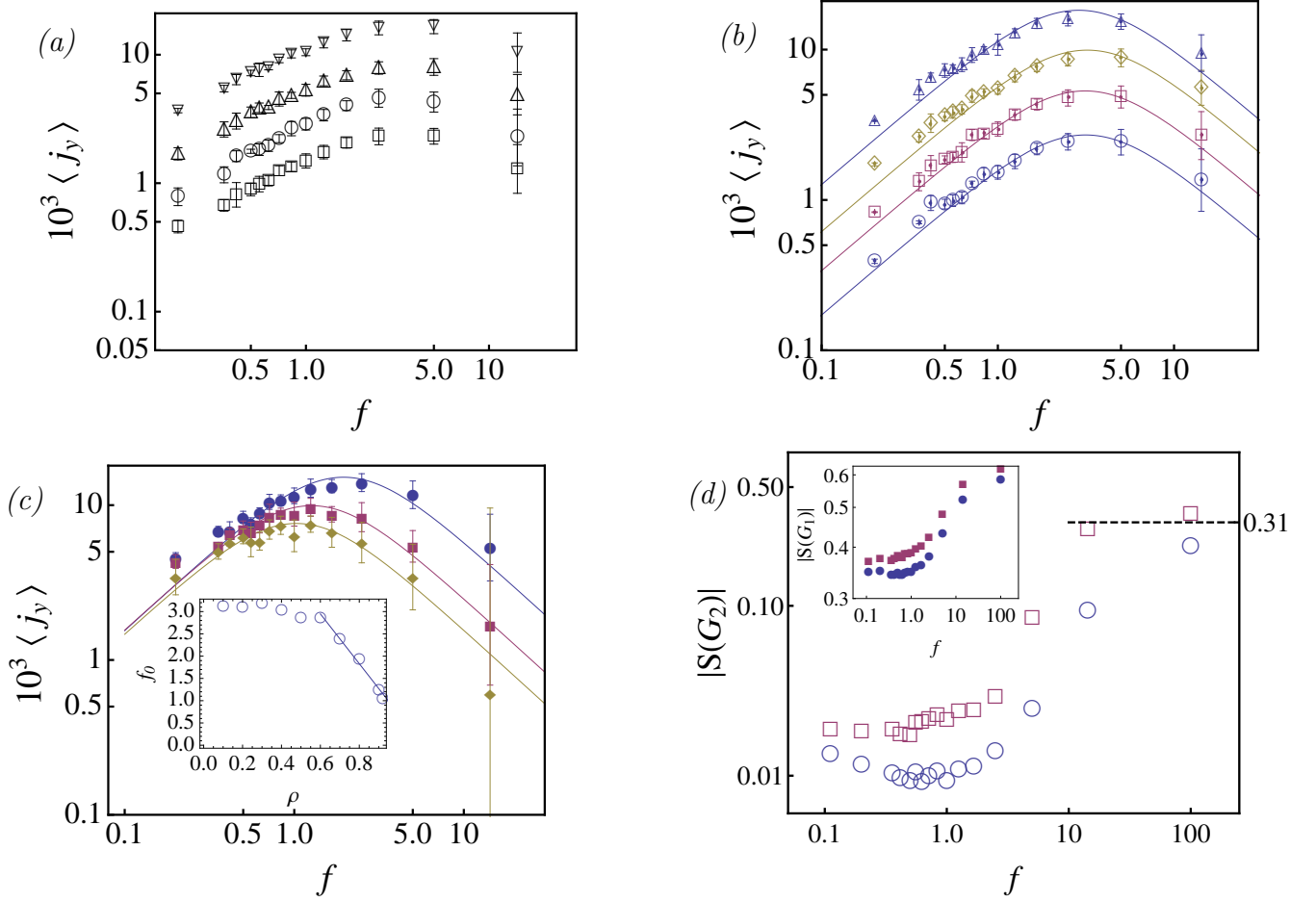


FIG. 7. (Color Online) (a) Plot of the averaged directed current for a non-interacting system driven by a periodic potential with periodicity 1σ , for densities $\rho = 0.1$ (\circ), 0.2 (\square), 0.3 (\diamond) and 0.6 (\triangle). The resonance in the measured flux occurs at a fixed frequency $f_0 = 3.5$. (b) Plot of the measured flux for an interacting system driven by a periodic potential with periodicity 1σ , for densities $\rho = 0.1$ (\circ), 0.2 (\square), 0.3 (\diamond) and 0.6 (\triangle). The solid lines are fit to the data using the functional form of Eq. (2) in the main text. (c) Plot of the measured flux for an interacting system driven by a periodic potential with periodicity 1σ , for densities $\rho = 0.8$ (\bullet), 0.92 (\blacksquare) and 0.98 (\blacklozenge). The solid lines are fit to the data using the functional form of Eq. (2) in the main text. The inset depicts the dependence of the resonance frequency on the density of the suspension. The solid line is a linear fit of $f_0 \sim (1 - \rho/\rho_c)$ to the data with $\rho_c \approx 1.13$. (d) Plot of the measured order parameter for the reciprocal lattice vectors \mathbf{G}_2 (empty symbols) and \mathbf{G}_1 (inset, filled symbols) for densities $\rho = 0.94$ (\bullet , \circ) and 0.96 (\blacksquare , \square).

travel a potential-valley of length λ follows the kinematic relation $\lambda \sim (U_0/\lambda)\tau_b^2$ leading to $\tau_b \sim \lambda/\sqrt{U_0}$, independent of density as λ itself is a constant. As a result the resonance frequency is also constant.

If, instead, an interacting system is driven by the same ratcheting potential with $\lambda = 1\sigma$, the resonance frequency remains approximately constant at lower densities ($\rho \lesssim 0.5$) indicating a free-particle like ballistic motion. However, at higher densities the resonance frequency drops approximately linearly with density (inset of Fig. 7 (c)). This indicates a cross-over from ballistic to diffusive transport with diffusive time-scale $\tau_D \sim \lambda^2/D_0(1 - \rho/\rho_c)$, and the corresponding resonance frequency $\sim (1 - \rho/\rho_c)$. This behavior should be contrasted against the $f \sim \rho(1 - \rho/\rho_c)$ behavior for *commensurately* ratcheting soft-disks at high densities (Fig.3 of main text). Note that the reasoning used here is similar to that we applied for interacting particles driven by a commensurate ratchet (See Eq.s (2)–(4) of main text).

It is known that, in case of laser induced freezing (LIF), the soft-core system of particles undergo a modulated liquid to solid transition near $\rho = 0.95$ and a periodic commensurate potential of strength $\beta U_0 = 1$ [9]. As we argued in the main text, at very high frequencies of weakly asymmetric commensurate ratcheting, one expects to recover fluid-solid transitions similar to LIF. In Fig. 7 (d) we present the dependence on the ratcheting frequency f of the solid order

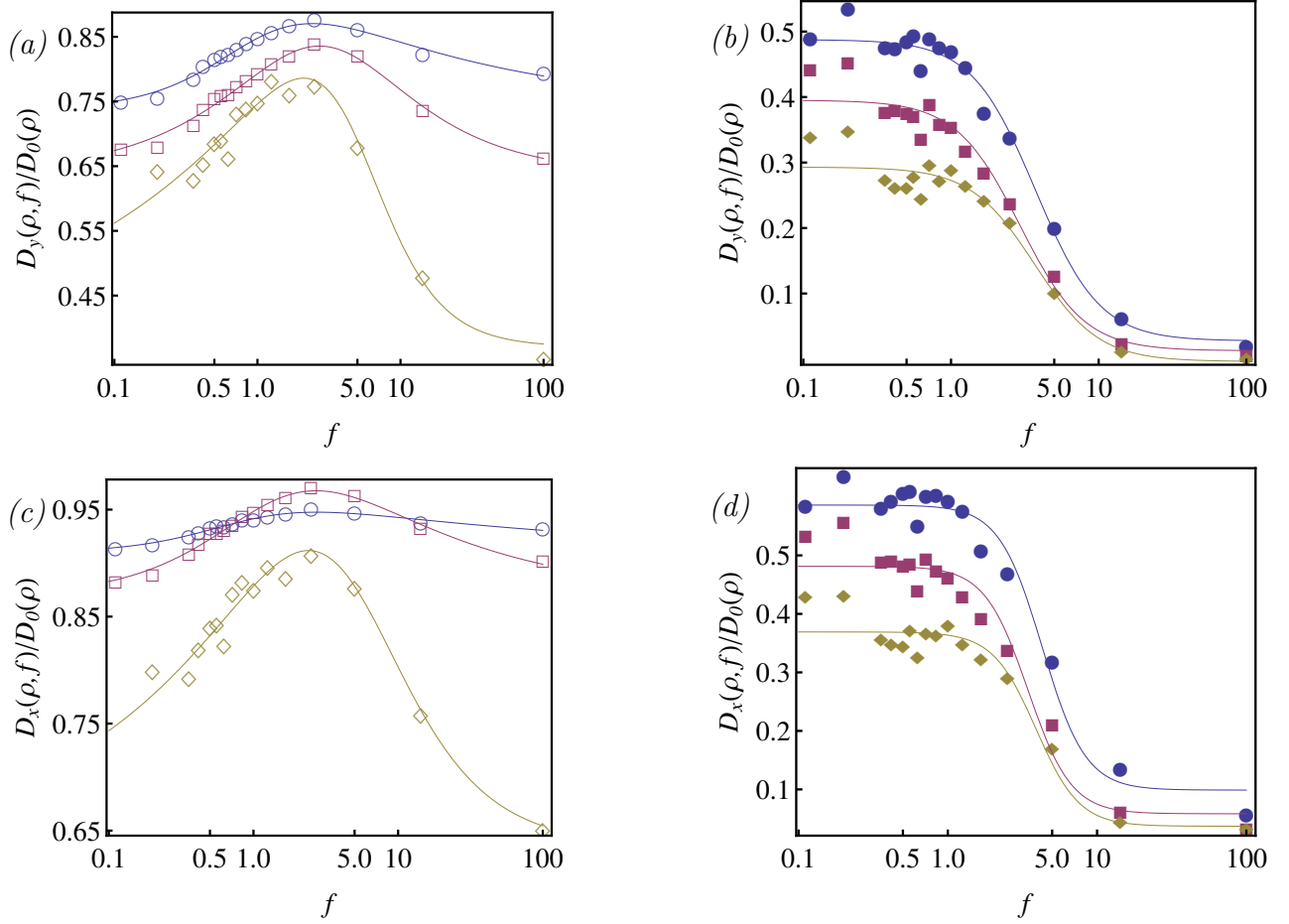


FIG. 8. Plot of the measured diffusion coefficient, normalized by the diffusion coefficient of a free system $D_0(\rho)$, for an interacting system driven by a periodic and a commensurate potential, along (in (a) and (b)) and perpendicular (in (c) and (d)) to the direction of the external drive, as a function of frequency for densities of the suspension $\rho = 0.2$ (\circ), 0.5 (\square), 0.8 (\diamond), 0.92 (\bullet), 0.94 (\blacksquare) and 0.96 (\blacklozenge). The solid lines are guide to the eye.

parameter $S(\mathbf{G}_2)$ (defined in main text) at densities $\rho = 0.94$ and 0.96 . This shows that at high enough frequencies $f \gg 10$, although the effective time-integrated periodic potential strength $\beta U_0 < 1$, the system at $\rho = 0.96$ already shows freezing, a reminiscent of LIF.

Finally, we consider the asymmetric diffusivity of the soft-core particle system, driven by a ratchet with periodicity commensurate with the density. We obtain the density and ratcheting-frequency dependence of the diffusivity. In Fig. 8, we show the frequency dependence of diffusion coefficients D_y , D_x along and perpendicular to the direction of ratcheting drive, respectively. We observe that $D_y < D_x$, since to diffuse in the direction of driving, particles have to climb potential barriers. As with the directed current, we also observe a resonance in the diffusion coefficient when the density is relatively small (see fig. 8(a) and (c)). The properties of $D_{x,y}(f, \rho)$ is further presented as a surface plot in Fig. 9, in the low density regime where the resonance structure is seen. The magnitude of both D_x and D_y increases with density, the effect being more pronounced for D_x and persists for densities lower than ≈ 0.3 (see Fig. 9), beyond which the magnitude of the diffusion constants steadily decreases. At higher densities, the resonance structure is lost, the diffusivities remain approximately constant at low frequencies, decreasing at frequencies $f \gtrsim 1$ (Figs 8(b) and (d)).

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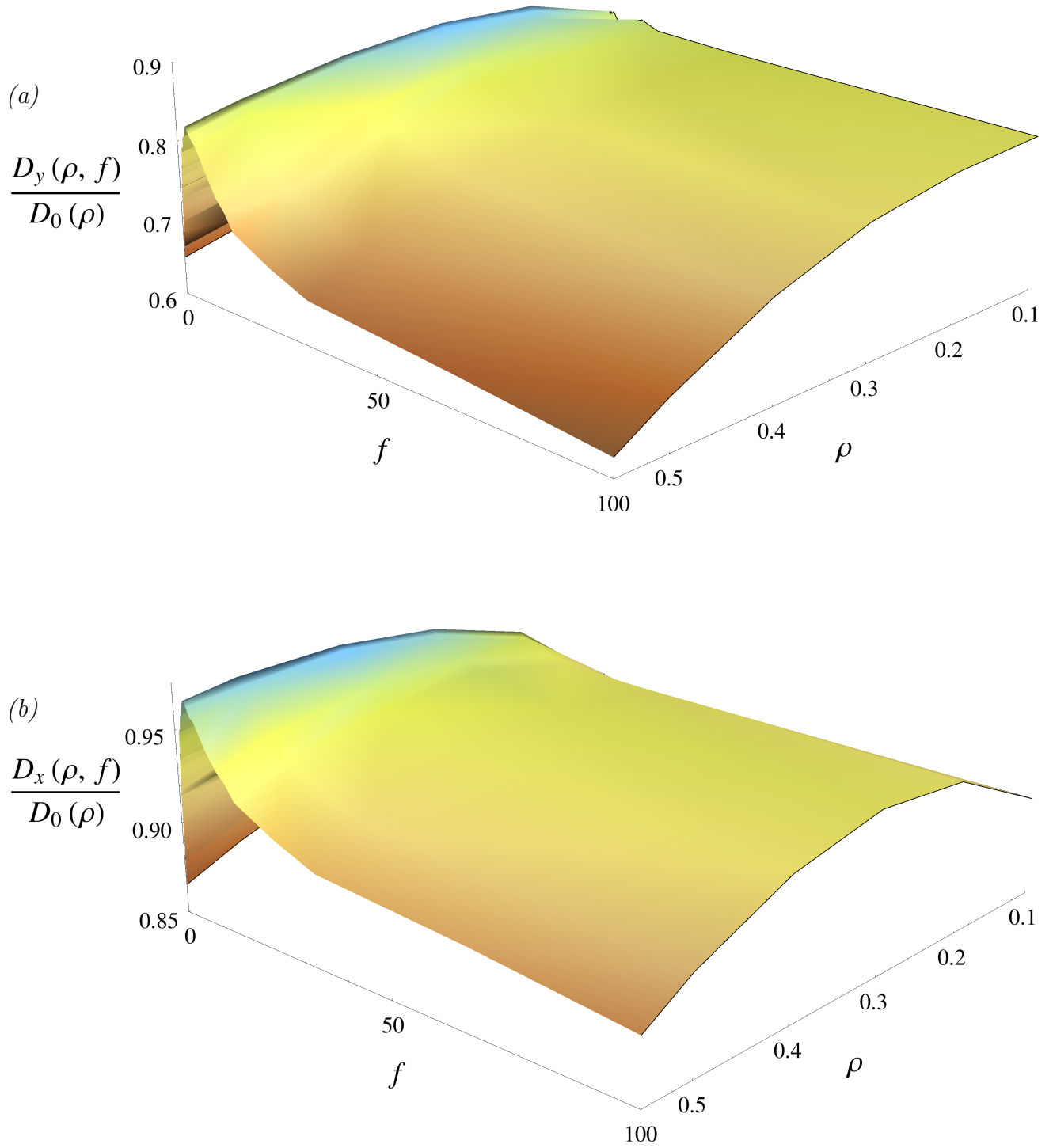


FIG. 9. Plot of the measured diffusion coefficient, normalized by the diffusion coefficient of a free system $D_0(\rho)$, for an interacting system driven by a periodic and a commensurate potential, along (a) and perpendicular (b) to the direction of the external drive, as a function of frequency and density.